4.2) Expanding $(a + bx)^n$

Worked example

Find first four terms in the binomial expansion of $\sqrt{2+x}$

State the values of x for which the expansion is valid.

Your turn

Find first four terms in the binomial expansion of $\sqrt{4+x}$

State the values of x for which the expansion is valid.

$$2+\frac{1}{4}x-\frac{1}{64}x^2+\frac{1}{512}x^3-\cdots$$

Valid for $|x|<4$

Worked example

Find first four terms in the binomial expansion of $\frac{1}{(3+2x)^4}$

State the values of x for which the expansion is valid.

Your turn

Find first four terms in the binomial expansion of $\frac{1}{(2+3x)^2}$

State the values of x for which the expansion is valid.

$$\frac{1}{4} - \frac{3}{4}x + \frac{27}{16}x^2 - \frac{27}{8}x^3 + \cdots$$

Valid for $|x| < \frac{2}{3}$

Worked example

Find first three terms in ascending powers of x of the series expansion of $\frac{3x+4}{\sqrt{2-5x}}$

State the values of x for which the expansion is valid.

Your turn

Find first three terms in ascending powers of x of the series expansion of $\frac{3x-4}{\sqrt{5+2x}}$

State the values of x for which the expansion is valid.

$$-\frac{4\sqrt{5}}{5}+\frac{19\sqrt{5}}{25}x+\frac{9\sqrt{5}}{25}x^2$$

Valid for $|x|<\frac{5}{2}$

Worked example

Use the binomial expansion of $\sqrt{8+9x}$ up to the x^2 term to estimate $\sqrt{11}$, giving your answer as a single fraction

Your turn

Use the binomial expansion of $\sqrt{9+8x}$ up to the x^2 term to estimate $\sqrt{11}$, giving your answer as a single fraction

$$\frac{179}{54}$$

Worked example

Find the series expansion, in ascending powers of x, up to and including the x^2 term for:

$$\frac{6}{2-3x} - \frac{4}{5+2x}$$

Your turn

Find the series expansion, in ascending powers of x, up to and including the x^2 term for:

$$\frac{6}{2+5x} - \frac{4}{3-2x}$$

$$\frac{5}{3} - \frac{151}{18}x + \frac{1961}{108}x^2$$

Worked example

Find the percentage error in approximating $\sqrt{53}$ using $x = \frac{1}{9}$ in the series expansion of $\sqrt{6 - x}$ up to and including the x^2 term.

Your turn

Find the percentage error in approximating $\sqrt{35}$ using $x = \frac{1}{9}$ in the series expansion of $\sqrt{4 - x}$ up to and including the x^2 term.

0.000138%

Worked example

State when the binomial expansion is valid:

$(2 + x)^{-3}$

$(9 + 2x)^{\frac{1}{2}}$

$(8 - x)^{\frac{1}{3}}$

$(5 - 2x)^{-3}$

$(16 + 3x)^{-\frac{1}{2}}$

Your turn

State when the binomial expansion is valid:

$(25 - 2x)^{-\frac{3}{4}}$

$|x| < \frac{25}{2}$